ISSN 1995-0802, Lobachevskii Journal of Mathematics, 2018, Vol. 39, No. 8, pp. 1099–1107. © Pleiades Publishing, Ltd., 2018.

Over-Determined Boundary Value Problem Method in the Theory of Mixed Problems for Acoustic Equations in Spherical Regions

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Received April 5, 2018

Abstract—The over-determined boundary value problem method is extended to solve some mixed problems for acoustic equations in spherical coordinates. The solvability conditions of auxiliary over-determined problems for the coordinate regions are obtained. These conditions are used to move from initial mixed boundary value problems to dual series equations of the standard form and then to infinite sets of linear algebraic equations.

DOI: 10.1134/S1995080218080073

Keywords and phrases: *Acoustic equations, spherical coordinates, over-determined boundary value problem.*

1. INTRODUCTION

It is well known that boundary value problems for spherical regions represent a very important class of problems in mathematical physics (see, for example, [1]). There are many publications dedicated to diffraction problems of acoustic, electromagnetic and elastic waves by spherical obstacles and to problems, in which eigen and forced oscillations in the spherical regions are sought. The modern theory of direct and inverse problems of wave scattering theory is considered in the books [2, 3]. Various approaches to solving acoustic problems in spherical coordinates are discussed in the monograph [4], as well as in works [5–10]. We mark the papers [11–15] as modern results.

From the practical point of view, the problems with mixed boundary conditions are most interesting and, at the same time, are least investigated. These problems arise in the theory of diffraction of acoustic and elastic wave by cracks or hard inclusions, and in the theory of diffraction of electromagnetic wave by thin conductive screens (see, for example, [16, 17]).

In the classical boundary value problems for acoustic equations, the values of pressure or normal component of the velocity are imposed on the boundary of a considered region. Transmission conditions should be imposed on the media interfaces; these conditions link the boundary values of unknown functions on two opposite sides of the boundary. The mixed problems for acoustic equations, when different boundary conditions or transmission conditions are imposed, become more complex. In this paper, mixed problems which have a concrete physical interpretation, are investigated.

The classical method of solving boundary value problems for wave equations in the spherical coordinates is based on the decomposition of the unknown field by the set of spherical waves obtained by method of separation of variables (see, for example, [18]). In this case, the initial problems are usually

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